

Multi-objective thermodynamic-based optimization of output power of Solar Dish-Stirling engine by implementing an evolutionary algorithm



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ABSTRACT

A solar-powered high temperature differential Stirling engine has been considered for optimization with multiple criteria. A mathematical model based on the finite-time thermodynamics has been developed so that the output power and thermal efficiency and the rate of entropy generation of the solar Stirling system with finite rate of heat transfer, regenerative heat loss, conductive thermal bridging loss and finite regeneration process time are obtained. Furthermore, imperfect performance of the dish collector and convective/radiative heat transfer mechanisms at the hot end as well as the convective heat transfer at the heat sink of the engine are considered in the developed model. Three objective functions including the output power and overall thermal efficiency have been considered simultaneously for maximization and the rate of entropy generation of the Stirling engine are minimized at the same time. Multi-objective evolutionary algorithms (MOEAs) based on NSGA-II algorithm has been employed while the Effectiveness's of regenerator, the Effectiveness's of the low temperature heat exchanger, the Effectiveness's of the high temperature heat exchanger, heat capacitance rate of the heat sink, heat capacitance rate of the heat source, temperatures of the working fluid in the high temperature isothermal process and temperatures of the working fluid in the low temperature isothermal process are considered as decision variables. Pareto optimal frontier has been obtained and a final optimal solution has been selected using various decision-making approaches including the fuzzy Bellman–Zadeh, LINMAP and TOPSIS methods.

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1. Introduction

The Stirling engine is a simple type of external-combustion engine that uses a compressible fluid as a working fluid. The Stirling engine can theoretically be a very efficient engine to convert heat into mechanical work at Carnot efficiency. The thermal limit for the operation of a Stirling engine depends on the material used for its construction. In most instances, the engines operate with a heater and cooler temperature of 923 and 338 K, respectively [1]. Engine efficiency ranges from about 30% to 40% resulting from a typical temperature range of 923–1073 K, and normal operating speed range from 2000 to 4000 rpm [2].

Urieli and Berchowitz [3], Reader and Hooper [4] and Hargreaves [5] among others all provide isothermal models similar to Schmidt's original work. Carlson et al. [6] developed an ideal model with non-isothermal heat exchange. Models of this type

suggest an improvement on the isothermal model as they eliminate the necessity for infinite heat transfer and impractically slow engine speed associated with isothermal working spaces. Urieli and Kushnir [7] showed that this analysis can be utilized in order to evaluate the various practical effects of non-ideal regenerators, heat exchangers, including heat transfer and pressure losses. Martaj et al. [8] presented a thermodynamic analysis of a low temperature Stirling engine at steady state operation, and energy, entropy and exergy balances were presented at each main element of the engine. The major goals of the Stirling engine designers can be mentioned in three categories: maximum efficiency; maximum power; minimum costs. Markman et al. [9] studied the thermal-flux and mechanical-power losses of a 200 W beta-configuration of the Stirling engine to optimize and increase the engine efficiency.

A miniature Stirling engine with 4 W output power in 0.1 MPa pressure and 900 rpm rotation speed was studied and made by Kagawa et al. [10]. Brandhorst and Chapman [11] developed a 5 kW engine for use as a power generator in space applications. Ataer [12] employed the Lagrangian method to the analysis of regenerators of Stirling cycle machines. The governing equations

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Nomenclature

A	area (m^2)	1,2,3,4	state points
C	heat capacitance rate (W K^{-1})	2	outlet
C_v	specific heat capacity ($\text{J mol}^{-1} \text{K}^{-1}$)	<i>app</i>	collector aperture
h	heat transfer coefficient ($\text{W m}^{-2} \text{K}^{-1}$)	<i>ave</i>	average
I	direct solar flux intensity (W m^{-2})	<i>c</i>	cold side
K_0	heat leak coefficient (W K^{-1})	<i>dis</i>	distance
M	proportionality constant (–)	H	heat source
n	number of mole	h	hot side
N	number of heat transfer units (–)	L	cold side/heat sink
P	power output (W)	R	regenerator
Q	heat (J)	<i>rec</i>	absorber
R	the gas constant ($\text{J mol}^{-1} \text{K}^{-1}$)		
S	entropy (J/K)	Greek	
T	temperature (K)	η	thermal efficiency [–]
t	time (s)	ε	effectiveness and emissivity factor [–]
U	overall heat transfer coefficient ($\text{W K}^{-1} \text{m}^{-2}$)	λ	ratio of volume during the regenerative processes [–]
V	volume of the working fluid (m^3)	δ	Stefan's constant [$\text{W m}^{-2} \text{K}^{-4}$]
W	output work (J)	σ	entropy production [W K^{-1}]
Subscripts			
0	ambient		
1	inlet		

of the regenerator are derived in terms of the displacement of the displacer, so that time does not appear in the equations. The equations, which include pressure fluctuations owing to flow reversals and longitudinal conduction, are solved numerically by a digital computer using a finite difference method. Nakajima et al. [13] developed a 10 g micro-Stirling engine with an approximately 0.05 cm^3 piston swept volume. An engine output power of 10 mW at 10 Hz was reported. The problems of scaling down were discussed.

Aramtummaphon [14] examined an open cycle Stirling engines by using steam heated from producer gas. The first engine generated an indicated power of about 1.36 kW at a maximum speed of 950 rpm, while the second engine, improved from the first one, produced an indicated power of about 2.92 kW at a maximum speed of 2200 rpm. Fukui et al. [15] has designed and constructed a micro-engine, and its experimental examination was performed; however, the micro-engine performance cannot be scaled to practical engine. Experimental determination of the effect of operating variables on Stirling engine is complicated and time consuming. Iwamoto et al. [16] comprised low temperature and high temperature Stirling engine efficiency with others. They determination exhibited that efficiency of LTD Stirling engine is around 50% of Carnot efficiency with same situation. Wu et al. [17] showed the effects of heat transfer, regeneration time, and imperfect regeneration on the performance of the irreversible Stirling engine cycle. Erbay and Yavuz [18] analyzed the real Stirling heat engine for maximum power output conditions using polytropic processes. They also determined the efficiency and compression ratio at maximum power density and ascertained the thermal design bounds. Ahmadi and Hosseinzade [19] investigated of Solar Collector Design Parameters Effect onto Solar Stirling Engine Efficiency. Ahmadi et al. [20] developed intelligent approach to figure power of solar Stirling heat engine by implementation of evolutionary algorithm.

Yaqi et al. and Sharma et al. developed a mathematical model for the overall thermal efficiency of solar powered high temperature differential dish Stirling engine with finite heat transfer and irreversibility of regenerator and optimized the absorber temperature and corresponding thermal efficiency [21,22]. Tili

investigated effects of regenerating effectiveness and heat capacitance rate of external fluids in heat source/sink at maximum power and efficiency [23]. Kaushik et al. studied the effects of irreversibilities of regeneration and heat transfer of heat/sink sources [24–27]. Solution of the multi-objective optimization problems is an extremely difficult goal which requires the simultaneous satisfaction of a number of different and even conflicting objectives. Evolutionary algorithms (EA) were initially extended and employed during the mid-eighties in an attempt to stochastically solve problems of this generic class [28]. A reasonable solution to a multi-objective problem is to investigate a set of solutions, each of which satisfies the objectives at an acceptable level without being dominated by any other solution [29]. Multi-objective optimization problems in general show a possibly uncountable set of solutions namely as Pareto frontier, whose evaluated vectors represent the best possible trade-offs in the objective function space. In this term, multi-objective optimization of different thermodynamic and energy systems have been paid attention by researchers nowadays [30–37]. In this work, by implementing multi-objective optimization algorithms output power and Stirling engine's thermal efficiency are maximized and rate of entropy generation of the Solar-Dish Stirling engine minimized. Also final solutions of different multi-objective optimization were compared with Ref. [24] data. At end it should be mentioned that, error analysis were implemented to figure robustness and precision of final solutions of different decision making approaches out.

2. System description

In Solar-Dish Stirling systems, mirrors of the parabolic shaped concentrator focuses the sun light on the focal point of the concentrator where the hot end of the Stirling engine is located. Therefore, the solar energy with a relatively high temperature is transferred to the hot side heat exchanger of the Stirling engine. Fig. 1 illustrates a schematic for a Solar-Dish Stirling engine connected to a solar dish concentrator. The Solar-Dish is equipped with a sun tracker which tracks the sun in order to have maximum solar energy transfer to the engine when sun moves during days. Hence,

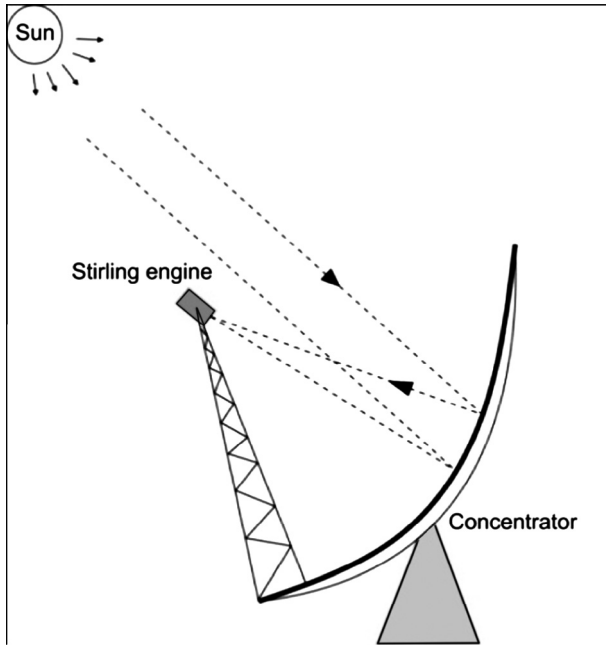


Fig. 1. Schematic of a solar Stirling engine [32].

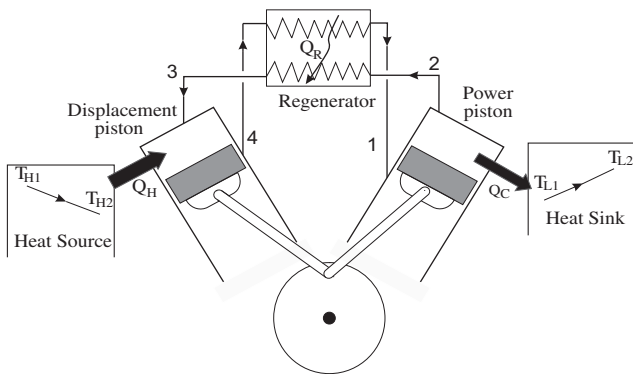


Fig. 2. Schematic diagram of the Stirling heat engine cycle.

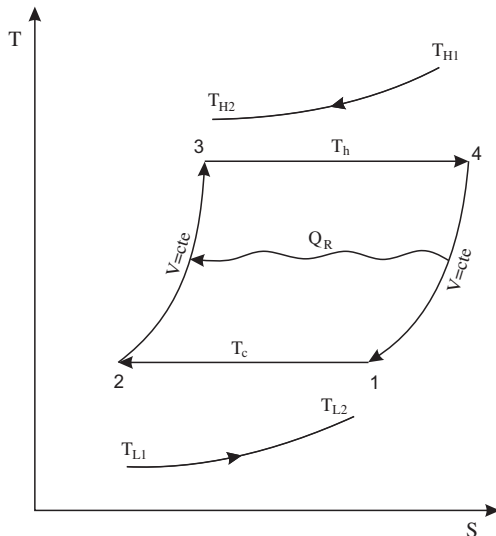


Fig. 3. T-S diagram of a Stirling engine cycle.

solar energy is absorbed and transferred to the working fluid in the hot space of engine.

Fig. 2 is a schematic diagram of a Stirling heat engine cycle with finite-time heat transfer and regenerative heat losses. As shown in Fig. 3, Stirling cycle consists of four processes. Process 1–2 is an isothermal process, in which the working fluid after compressing at constant temperature, T_c and rejected heat to the heat sink at low temperature T_{L1} . Therefore, the temperature of heat sink is increased to T_{L2} . Then the working fluid crosses over the regenerator and warms up to T_h in an isochoric process 2–3. In process 3–4, the working fluid is expanded in a constant temperature T_h process and receives heat from the heat source in which its temperature is reduced from T_{H1} into T_{H2} . Last process (4–1), is an isochoric cooling process, where the regenerator absorbs heat from the working fluid.

3. Thermodynamic analysis of the system

Actual useful heat gain of the dish collector, considering conduction, convection and radiation losses is given by [20–22] as follows:

$$q_u = IA_{app}\eta_0 - A_{rec}[h(T_{H_{ave}} - T_0) + \varepsilon\delta(T_{H_{ave}}^4 - T_0^4)] \quad (1)$$

where I is the direct solar flux intensity, A_{app} is the collector aperture area, η_0 is the collector optical efficiency, A_{rec} is the absorber area, h is conduction/convection heat transfer coefficient, $T_{H_{ave}}$ is the average absorber temperature, T_0 is the ambient temperature, ε is emissivity factor of the collector, δ is the Stefan's constant.

Thermal efficiency η_s of the dish collector is obtained as [20–22]:

$$\eta_s = \frac{q_u}{IA_{app}} = \eta_0 - \frac{1}{IC}[h(T_{H_{ave}} - T_0) + \varepsilon\delta(T_{H_{ave}}^4 - T_0^4)] \quad (2)$$

3.1. Regenerative heat losses in the regenerator

It is important to mention that there also exists a finite heat transfer in the regenerative heat transfer (Q_R) which is given by [20–22]

$$Q_R = nC_v\varepsilon_R(T_h - T_c) \quad (3)$$

Where ΔQ_R is the heat loss during the two regenerative processes in the cycle. The following relationship is obtained [20–22]:

$$\Delta Q_R = nC_v(1 - \varepsilon_R)(T_h - T_c) \quad (4)$$

n is the mass of the working fluid in mole, C_v is the specific heat capacity of the working fluid in the regenerative processes in terms of mole, ε_R is the effectiveness of regenerator, T_h and T_c are the working fluid temperatures in the hot space and cold space, respectively.

Owing to the influence of irreversibility of the finite-rate heat transfer, the time of the regenerative processes is not negligible in comparison to that of the two isothermal processes [20]. In order to calculate the time of the regenerative processes, one assumes that the temperature of the working fluid in the regenerative processes as a function of time is given by [20]:

$$\frac{dT}{dt} = \pm M_i \quad (5)$$

where M is the proportionality constant which is independent of the temperature difference and dependent only on the property of the regenerative material, called regenerative time constant and the $\pm$ sign belong to the heating ($i = 1$) and cooling ($i = 2$) processes respectively [20].

$$t_3 = \frac{T_1 - T_2}{M_1} \quad (6)$$

$$t_4 = \frac{T_1 - T_2}{M_2} \quad (7)$$

3.2. The amounts of heat released by the heat source and absorbed by the heat sink

The heat released between heat source and working fluid (Q_h), the heat absorbed between the working fluid and the heat sink (Q_c) is obtained as follows

$$Q_h = nRT_h \ln \lambda + nC_v(1 - \varepsilon_R)(T_h - T_c) \quad (8)$$

$$Q_c = nRT_c \ln \lambda + nC_v(1 - \varepsilon_R)(T_h - T_c) \quad (9)$$

In the other hand:

$$Q_h = [C_H \varepsilon_H (T_{H_1} - T_h) + \xi C_H \varepsilon_H (T_{H_1}^4 - T_h^4)] t_h \quad (10)$$

$$Q_c = C_L \varepsilon_L (T_c - T_{L_1}) t_l \quad (11)$$

where C_H and C_L are the heat capacitance rate of external fluids in the heat source and heat sink, respectively.

$$\varepsilon_H = 1 - e^{-N_H} \quad (12)$$

$$\varepsilon_L = 1 - e^{-N_L} \quad (13)$$

where ε_H and ε_L are the Effectiveness's of the high and low temperature heat exchangers, respectively. And $N_L = U_L A_L / C_L$, $N_H = U_H A_H / C_H$ are the cyclic period. Using Eqs. (3)–(11), we get that the cyclic period t is:

$$t = \frac{nRT_h \ln \lambda + nC_v(1 - \varepsilon_R)(T_h - T_c)}{C_H \varepsilon_H (T_{H_1} - T_h) + \xi C_H \varepsilon_H (T_{H_1}^4 - T_h^4)} + \frac{nRT_c \ln \lambda + nC_v(1 - \varepsilon_R)(T_h - T_c)}{C_L \varepsilon_L (T_c - T_{L_1})} + \left(\frac{1}{M_1} + \frac{1}{M_2} \right) (T_h - T_c) \quad (14)$$

3.3. The conductive thermal bridging losses from heat source to the heat sink

This value is proportional to the average temperature difference of the heat source and heat sink and the cycle time, it is obtained as follows:

$$Q_o = K_o (T_{H_{ave}} - T_{L_{ave}}) t_{cycle} \quad (15)$$

$$T_{H_{ave}} = \frac{T_{H_1} + T_{H_2}}{2} \quad (15a)$$

$$T_{L_{ave}} = \frac{T_{L_1} + T_{L_2}}{2} \quad (15b)$$

We have [24–27]

$$T_{H_2} = (1 - \varepsilon_H) T_{H_1} + \varepsilon_H T_h \quad (15c)$$

$$T_{L_2} = (1 - \varepsilon_L) T_{L_1} + \varepsilon_L T_c \quad (15d)$$

Thus, using Eqs. (15a)–(15d), we have

$$Q_o = \frac{K_o}{2} [(2 - \varepsilon_H) T_{H_1} - (2 - \varepsilon_L) T_{L_1} + (\varepsilon_H T_h - \varepsilon_L T_c)] t_{cycle} \quad (16)$$

The net heat released from the heat source (Q_H) and the net heat absorbed by the heat sink (Q_L) are obtained as follows:

$$Q_H = Q_h + Q_o \quad (17)$$

$$Q_L = Q_c + Q_o \quad (18)$$

Considering the cyclic period of the Stirling engine, the output power, the thermal efficiency and entropy production of the engine are given by:

$$P = \frac{W}{t} = \frac{Q_H - Q_L}{t} \quad (19)$$

$$\eta_t = \frac{Q_H - Q_L}{Q_H} \quad (20)$$

$$\sigma = \frac{1}{t} \left(\frac{Q_L}{T_{L_{ave}}} - \frac{Q_H}{T_{H_{ave}}} \right) \quad (21)$$

Substituting Eqs. (3)–(14) into Eqs. (19) and (20) we have,

$$P = \frac{nR(T_h - T_c) \ln \lambda}{\frac{nRT_h \ln \lambda + nC_v(1 - \varepsilon_R)(T_h - T_c)}{C_H \varepsilon_H (T_{H_1} - T_h) + \xi C_H \varepsilon_H (T_{H_1}^4 - T_h^4)} + \frac{nRT_c \ln \lambda + nC_v(1 - \varepsilon_R)(T_h - T_c)}{C_L \varepsilon_L (T_c - T_{L_1})} + \left(\frac{1}{M_1} + \frac{1}{M_2} \right) (T_h - T_c)} \quad (22)$$

$$\eta_t = \frac{nR(T_h - T_c) \ln \lambda}{nRT_h \ln \lambda + nC_v(1 - \varepsilon_R)(T_h - T_c) + \frac{K_o}{2} [(2 - \varepsilon_H) T_{H_1} - (2 - \varepsilon_L) T_{L_1} + (\varepsilon_H T_h - \varepsilon_L T_c)] t_{cycle}} \quad (23)$$

The maximum thermal efficiency of the entire Solar-Dish Stirling engine is product of the thermal efficiency of the collector and the optimal thermal efficiency of the Stirling engine [22]. Namely:

$$\eta_m = \eta_s \eta_t \quad (24)$$

4. Multi-objective optimization with evolutionary algorithms

4.1. Optimization via EA

In this study, the Pareto frontier is obtained using the Genetic Algorithm (GA) known as a branch of evolutionary algorithm. Genetic algorithms were developed by John Holland in the 1960s as a means of importing the mechanisms of natural adaptation into computer algorithms and numerical optimization [31]. They are implemented as a computer simulation in which a population of abstract representations (called chromosomes or the genotype of the genome) of candidate solutions (called individuals, creatures, or pheno-types) to an optimization problem evolves toward better solutions. The evolution normally starts from a population of randomly generated individuals and happens in generations. In each generation, the fitness of every individual in the population is evaluated; multiple individuals are stochastically selected from the current population (based on their fitness), and modified (recombined and possibly randomly mutated) to form a new population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations have been produced, or a satisfactory fitness level has been reached for the population. If the algorithm has terminated due to a maximum number of generations, a satisfactory solution may or may not have been reached. In genetic algorithms, a candidate solution to a problem is typically called a chromosome, and the evolutionary viability of each chromosome is given by a fitness function. This method is a powerful optimization tool for nonlinear problems [29,32].

Also, Multi-objective evolutionary algorithms (MOEAs) have been developed over the past decade by numerous tests on complex mathematical problems and on real-world engineering problems and have shown that they can eliminate the difficulties of classical methods [29,32]. The structure of the MOEA used in the present study is illustrated in Fig. 4 [31]. The real values of decision variables are used instead of their binary coded.

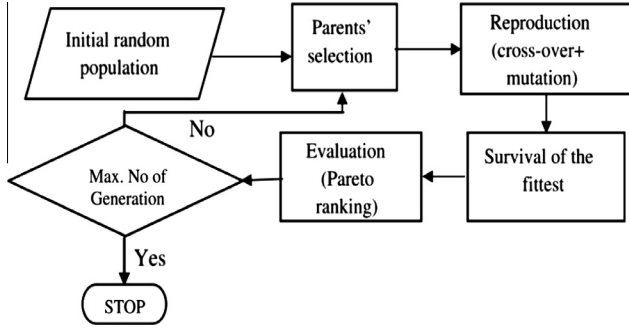


Fig. 4. Scheme for the multi-objective evolutionary algorithm used in the present study [31–33].

4.2. Non-dominated sorting genetic algorithm, NSGA-II

NSGA-II algorithm is utilized here as a multi-objective optimization method in order to find the Pareto frontier using the genetic algorithm. In this algorithm solutions are categorized based on the Pareto concept and sorting non-dominated solutions into non-dominated layers as schematically shown in Fig. 5. In other words, if N_p is the number of population, it is categorized into N_l layers in which intersection of each two arbitrary selected layers is empty set and union of all layers is N_p set.

Elitism or virtual fitness of each individual is equal to its layer. Parent selection for cross over operation is performed based on the tournament selection between two random selected layers. Therefore, individual located on the layer 1, have more chance to be selected for the next generation. Uniform distribution of solutions along layers is controlled with introducing of crowding distance index for each solution. This index is defined as a ratio of subtraction of objective functions for two neighbor solutions around the current solution to the subtraction of the maximum and minimum values of that objective. Therefore, for k th objective of j th solution, we have,

$$i_{dis,j,k} = \frac{f_{k,j-1} - f_{k,j+1}}{f_{k,max} - f_{k,min}} \quad (25)$$

For boundary solutions (solutions with smallest and largest function values) are assigned an infinite distance index. The overall crowding distance value is calculated as the sum of individual distance values corresponding to each objective.

$$I_{dis,j} = \sum_{k=1}^M i_{dis,j,k} \quad (26)$$

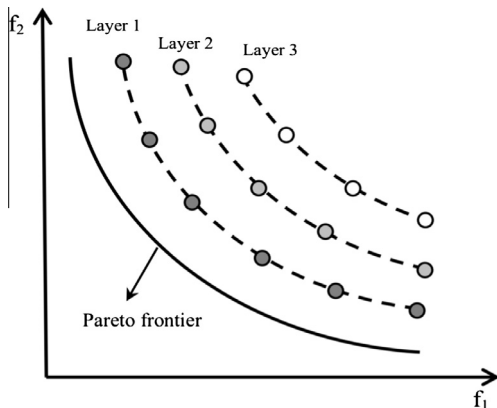


Fig. 5. Schematic of solution layering in NSGA-II algorithm [33].

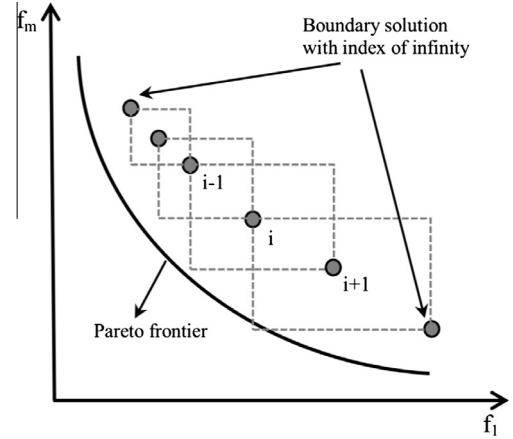


Fig. 6. Schematic of distance indexing of individuals in NSGA-II algorithm [33].

where M is the number of objectives and j is the individual index. Fig. 6 shows schematic of evaluation of distance index. In this algorithm, two parameters are calculated for each solution:

- (1) Dominant number, NL , that is the number of solutions that dominate the current solution. Detail of dominance concept and definition is explained well in multi-objective optimization in Refs. [28,30]. Dominant number, for non-dominated individuals of the current population is zero, therefore, these solutions are placed in layer 1. Non-dominated solutions for a set of the individuals excluding the layer 1 members are placed in layer 2. For an M objectives problem with N populations, the number of comparisons is MN^2 . This procedure continued so that all individuals are accommodated in their appropriate layers. In the continuing, $irank$ index for each individual is assigned as its dominant number (layer number), NL .
- (2) Crowded comparison operator, $n \prec$, defined as follow:

$$A \prec B \text{ if } (rank_A < rank_B) \text{ Or } ((rank_A = rank_B) \text{ and } I_{dis,A} > I_{dis,B}) \quad (27)$$

It means that for two individuals with differing non-dominated ranks (different layers), the solution with the lower rank (layer) is preferred. Otherwise, for two solutions of the same layer, the solution placed in the region with lower concentration of solutions is selected. This is the “concept of dominated sorting algorithm.”

4.3. Objective functions, decision variables and constraints

Three key objective functions for optimization are the system's entropy production (should be minimized), the output power (should be maximized) and the thermal efficiency of Dish-Stirling engine system's (should be maximized), denoted by Eqs. (21), (22) and (24), respectively.

In this paper, seven decision variables have been considered as follow:

ε_R is the Effectiveness's of regenerator; ε_L the Effectiveness's of the low temperature heat exchanger; ε_H the Effectiveness's of the high temperature heat exchanger; C_L the heat capacitance rate of the heat sink; C_H the heat capacitance rate of the heat source; T_h the temperatures of the working fluid in the high temperature isothermal process 3–4; and T_c the temperatures of the working fluid in the low temperature isothermal process 1–2.

The objective functions with respect to following constraints have been solved:

$$0.4 \leq \varepsilon_R \leq 0.9 \quad (28)$$

$$0.4 \leq \varepsilon_H \leq 0.8 \quad (29)$$

Table 1
Specified genetic algorithm options for optimization problem in this study.

Specified options for GA	Value
Population type	Duble vector
Population size	400
Selection process	Tournament
Tournament size	2
Mutation	Constraint dependent
Maximum number of generations	1000

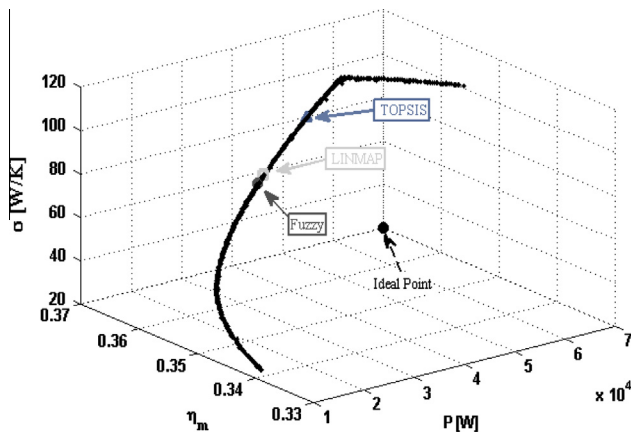


Fig. 7. Pareto optimal frontier in objectives' space.

$$0.4 \leq \varepsilon_L \leq 0.8 \quad (30)$$

$$300 \leq C_H \leq 1800 \quad (31)$$

$$300 \leq C_L \leq 1800 \quad (32)$$

$$800 \leq T_h \leq 1000 \text{ K} \quad (33)$$

$$400 \leq T_c \leq 510 \text{ K} \quad (34)$$

To find the optimum design parameters of the system, based on genetic algorithm technique a simulation program is developed in Matlab software. Genetic algorithm options for optimization problem are listed in Table 1.

4.4. Decision-making in the multi-objective optimization

In multi-objective optimization a process of decision-making for selection of the final optimal solution from available solutions is required. A number of several methods for decision-making process in decision problems exist. And these methods can be employed in decision-making for selection of a final optimal solution from the Pareto frontier which is obtained by optimization. In this paper most recognized and common type of decision-making processes including the fuzzy Bellman–Zadeh, LINMAP and TOPSIS method is used in parallel and the final optimal solution has been decided

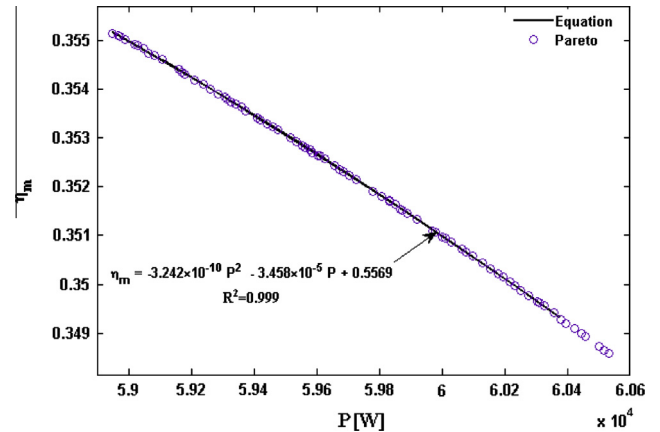


Fig. 8. Pareto optimal frontier in objectives' space ($\eta_m - P$).

based on engineering experience and criteria among solutions which suggested by these three methods. For more information about these methods see Refs. [38,39].

5. Result and discussion

The output power and thermal efficiency of Solar Dish–Stirling system are maximized simultaneously while the rate of entropy generation of the Stirling engine are minimized simultaneously using the multi-objective optimizing method which works based on the NSGA-II algorithm. In this regard, optimization is performed with objective functions that are expressed by Eqs. (21), (22) and (24), also constraints are expressed with Eqs. (28)–(34). Design parameters (decision variables) of optimization are the Effectiveness's of regenerator, Effectiveness's of the low temperature heat exchanger, Effectiveness's of the high temperature heat exchanger, heat capacitance rate of the heat sink, heat capacitance rate of the heat source, temperatures of the working fluid in the high temperature isothermal process 3–4, temperatures of the working fluid in the low temperature isothermal process 1–2. In order to have consistency with previous works, specifications of the Stirling engine are considered as follows [22,24],

$$\begin{aligned} I &= 1000 \text{ W/m}^2, \quad C = 1300, \quad \varepsilon = 0.9, \quad \eta_0 = 0.9, \quad K_0 = 2.5 \text{ W/K}, \\ n &= 1, \quad C_v = 15 \text{ J mol}^{-1} \text{ K}^{-1}, \quad R = 4.3 \text{ J mol}^{-1} \text{ K}^{-1}, \quad T_{H1} = 1300 \text{ K}, \\ T_{L1} &= 290 \text{ K}, \quad T_0 = 288 \text{ K}, \quad \xi = 2 \times 10^{-10}, \quad h = 20 \text{ W m}^{-2} \text{ K}^{-1}, \\ \lambda &= 2, \quad 1/M_1 + 1/M_2 = 2 \times 10^{-5} \text{ s/K}, \quad \delta = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}. \end{aligned}$$

Fig. 7 illustrates the Pareto frontier in the proposed objectives' space obtained using the multi-objective evolutionary algorithm (NSGA-II). Three final solutions selected by the Fuzzy Bellman–Zadeh, LINMAP and TOPSIS decision-makers are indicated separately in Fig. 7.

Table 2 compares optimal results obtained in this paper using three decision making methods with corresponding results obtained in Ref. [24]. Where Table 1 demonstrated obtained results

Table 2
Decision making of multi-objective optimal solutions.

Optimal solution	Decision making method	Decision variables							Objective functions		
		ε_R	ε_H	ε_L	C_H	C_L	T_h (K)	T_c (K)	η_m (%)	P (kW)	σ (W K ^{−1})
This work	Bellman–Zadeh	0.9	0.8	0.8	1410	774	996.7	400.5	36.33	37.37	70
	LINMAP	0.9	0.8	0.8	1413	823	996.7	400.5	36.36	38.86	69.5
	TOPSIS	0.9	0.8	0.8	1424	1252	996.9	400.5	36.56	49.64	87.47
Ref. [24]	–	0.9	0.8	0.8	1000	1000	962.2	462.2	34.2	69.97	–

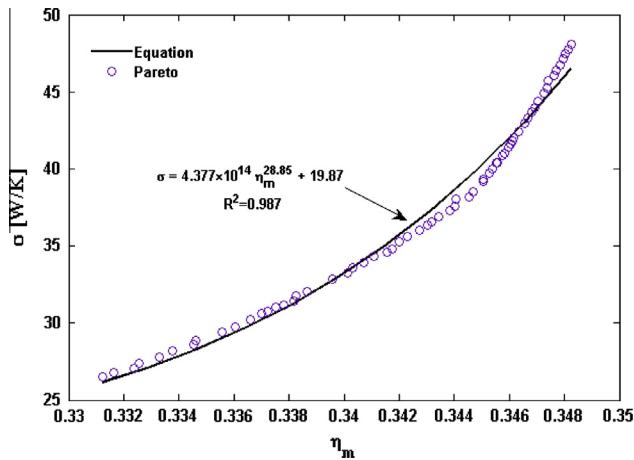


Fig. 9. Pareto optimal frontier in objectives' space ($\sigma - \eta_m$).

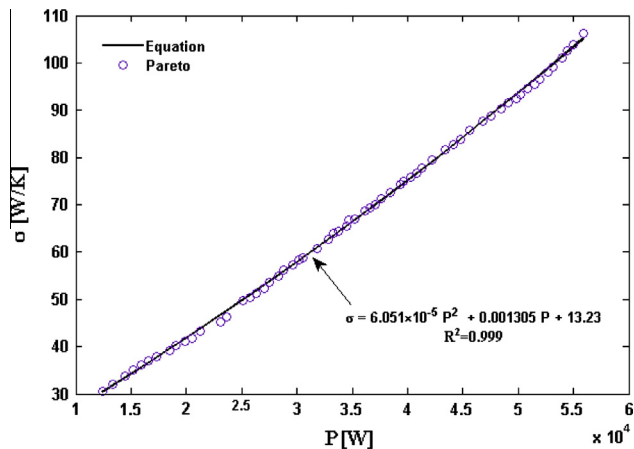


Fig. 10. Pareto optimal frontier in objectives' space ($\sigma - P$).

from different decision making approaches for Solar Dish-Stirling engine and results exhibited by Kaushik [24].

In Fig. 8 plot of the Pareto optimal frontier for objective functions of output power and Solar Dish-Stirling system's thermal efficiency which final solutions determined by different decision making approaches are highlighted. As can be seen in Fig. 8, the output power decreases with increasing the thermal efficiency of the engine. To assist the optimal design of the Solar Dish-Stirling system, fitted curve ($\eta_m = -3.242 \times 10^{-10} P^2 - 3.458 \times 10^{-5} P + 0.5569$), which is valid in the range of $59 \text{ kW} < P < 60.5 \text{ kW}$, is derived for the Pareto optimal points curve.

A previous analogue was carried out to illustrate Pareto optimal frontier for objective functions of Solar Dish-Stirling system's thermal efficiency and the rate of entropy generation of the Stirling engine. Output results from this analogue are demonstrated in Fig. 9 in which by increasing the thermal efficiency, rate of entropy generation increases. Also, a curve is fitted

($\sigma = 4.377 \times 10^{14} \eta_m^{28.85} + 19.87$) to all the points obtained by multi-objective optimization. In Fig. 10 Pareto optimal frontier for objective functions of output power and the rate of entropy generation of the Stirling engine is depicted. From Fig. 10 can be observed that, the rate of entropy generation of the engine increases when the output power increases. Moreover, a curve is fitted ($\sigma = 6.051 \times 10^{-5} P^2 + 0.001305 P + 13.23$) to the points obtained by multi-objective optimization.

Mean absolute percentage error was carried out in error analysis of implemented methods. To assess this goal, 30 runs of each approach were performed to get final solution by Bellman-Zadeh, LINMAP and TOPSIS decision making methods. Then, magnitude of each objective function such as output power, Solar Dish-Stirling system's thermal efficiency and rate of entropy generation of the Stirling engine were compared with correspond experimental values. First row of Table 3 demonstrates maximum absolute percentage error (MAAE) of three decision making methods. Moreover, second row of Table 3 reports mean absolute percentage error (MAPE) of mentioned methods.

6. Conclusion

In this study, finite-time thermodynamics has been applied to determine output power, thermal efficiency and rate of entropy generation of the Solar Dish-Stirling engine. Factors such as dish collector performance, conductive and radiative heat transfer mechanism at heat source and heat sink of the engine are involved in the analysis. The output power, thermal efficiency and rate of entropy generation of the engine were considered simultaneously for multi-objective optimization the Effectiveness's of regenerator (ε_R), the Effectiveness's of the low temperature heat exchanger (ε_L), the Effectiveness's of the high temperature heat exchanger (ε_H), heat capacitance rate of the heat sink (C_L), heat capacitance rate of the heat source (C_H), temperatures of the working fluid in the high temperature isothermal process (T_H) and temperatures of the working fluid in the low temperature isothermal process (T_L) were considered as design parameters. Multi objective evolutionary algorithm is considered based on NSGA-II algorithm and the Pareto optimal frontier in objectives space was obtained. A final optimal solution was selected from solutions of the Pareto frontier using three decision making methods including LINMAP, TOPSIS and fuzzy methods.

It was shown that multi objective-multi variable approach leads to more desirable design of engine if it is compared with traditional single objective-single variable approach. The priority of the multi objective-multi variable approach over the traditional single objective-single variable was proven as in all cases significant lower deviation indexes was obtained for multi objective-multi variable approach. Further in this case it was observed that although the TOPSIS decision-making method yields a final solution with more desirable output power and thermal efficiency, the rate of entropy generation is increased. Error analysis was performed based on the MAPE method and it was shown that the average error of solutions obtained through three decision making methods were 1.3%, 4.4% and 3.5% for the output power, thermal efficiency and rate of entropy generation, respectively. The maximum error of solutions obtained by three decision making methods were 2.5%, 8.4% and

Table 3

Error analysis based on the mean absolute percent error (MAPE) method.

Decision making method	Bellman-Zadeh			LINMAP			TOPSIS		
	η_m (%)	P (kW)	σ (W K ⁻¹)	η_m (%)	P (kW)	σ (W)	η_m (%)	P (kW)	σ (W K ⁻¹)
Max Error (%)	2.5	6.8	6.2	2.5	6.5	6.8	2.4	8.4	6.5
Average Error (%)	1.3	2.4	3.4	1.2	2.7	3.0	1.3	4.4	3.5

6.8% for the output power, thermal efficiency and rate of entropy generation, respectively.

References

- [1] Iskander T, Musmar SA. Thermodynamic evaluation of a second order simulation for Yoke Ross Stirling engine. *Energy Convers Manage* 2013;68: 149–60.
- [2] Formosa F, Despesse G. Analytical model for Stirling cycle machine design. *Energy Convers Manage* 2010;51(10):1855–63.
- [3] Urieli I, Berchowitz D. Stirling cycle engine analysis. Bristol: Adam Hilger; 1984.
- [4] Reader GT, Hooper C. Stirling engines. London: E. & F.N. Spon; 1983.
- [5] Hargreaves CM. The philips Stirling engine. Amsterdam: Elsevier Science Publishers B.V.; 1991.
- [6] Carlson H, Commisso MB, Lorentzen B. Maximum obtainable efficiency for engines and refrigerators based on the Stirling cycle. In: *Energy conversion engineering conference*, 1990. IECEC-90. Proceedings of the 25th intersociety; 1990.
- [7] Urieli I, Kushnir M. The ideal adiabatic cycle—a rational basis for Stirling engine analysis, 17th IECEC; 1982. p. 162–8.
- [8] Martaj N, Grosu L, Rochelle P. Thermodynamic study of a low temperature difference Stirling engine at steady state operation. *Int J Thermodyn* 2007;10(4):165–76.
- [9] Markman MA, Shmatok YI, Krasovskii VG. Experimental investigation of a low-power Stirling engine. *Geliotekhnika* 1983;19:19–24.
- [10] Kagawa N, Araoka K, Sakuma T, Ichikawa S. Design and development of a miniature Stirling engine. In: *The 25th international energy conversion engineering conference*, Reno, NV, USA; 6; 1990. p. 442–7.
- [11] Brandhorst Jr HW, Chapman Jr PA. New 5 kW free-piston Stirling space convertor developments. *Acta Astronaut* 2008;63(1–4):342–7.
- [12] Ataer OE. Numerical analysis of regenerators of piston type Stirling engines using Lagrangian formulation. *Int J Refrig* 2002;25:640–52.
- [13] Nakajima N, Ogawa K, Fujimasa I. Study on micro engines: miniaturizing Stirling engines for actuators. *Sens Actuator* 1989;20:75–82.
- [14] Aramtummaphon D. A study of the feasibility of using heat energy from producer gas for running Stirling engine by steam as working fluid. Master thesis, King Mongkut's University of Technology Thonburi; 1996.
- [15] Fukui T, Shiraishi T, Murakami T, Nakajima N. Study on high specific power micro-Stirling engine. *JSME Int Fluids Thermal Eng B* 1999;42(4):776.
- [16] Iwamoto I, Toda K, Hirata K, Takeuchi M, Yamamoto T. Comparison of low and high temperature differential Stirling engines. In: *Proceedings of the 8th international Stirling engine conference*, Ancona, Italy; 1997. p. 29–38.
- [17] Wu F, Chen L, Wu C, Sun F. Optimum performance of irreversible Stirling engine with imperfect regeneration. *Energy Convers Manage* 1998;39(8): 727–32.
- [18] Erbay LB, Yavuz H. Analysis of Stirling heat engine at maximum power conditions. *Energy* 1997;22(7):645–50.
- [19] Ahmadi MH, Hosseinzade H. Investigation of solar collector design parameters effect onto solar Stirling engine efficiency. *Appl Mech Eng* 2012;1:1–4.
- [20] Ahmadi MH, GhareAghaj SS, Nazeri A. Prediction of power in solar stirling heat engine by using neural network based on hybrid genetic algorithm and particle swarm optimization. *Neural Comput & Applic* 2013;22:1141–50.
- [21] Sharma A, Shukla SK, Rai AK. Finite time thermodynamic analysis and optimization of Solar-Dish Stirling heat engine with regenerative losses. *Therm Sci* 2011;15(4):995–1009.
- [22] Yaqi L, Yaling H, Weiwei W. Optimization of solar-powered Stirling heat engine with finite-time thermodynamics. *Renew, Energy* 2011;36(1):421–7.
- [23] Tlili I. Finite time thermodynamic evaluation of endoreversible Stirling heat engine at maximum power conditions. *Renew Sustain Energy Rev* 2012;16(4):2234–41.
- [24] Kaushik SC, Kumar S. Finite time thermodynamic evaluation of irreversible Ericsson and Stirling heat engines. *Energy Convers Manage* 2001;42:295–312.
- [25] Kaushik SC, Kumar S. Finite time thermodynamic analysis of endoreversible Stirling heat engine with regenerative losses. *Energy* 2000;25:989–1003.
- [26] Kaushik SC, Tyagi SK, Mohan S. Performance evaluation of irreversible Stirling and Ericsson heat engines. *Int J Ambient Energy* 2003;24:149–56.
- [27] Tyagi SK, Kaushik SC, Salhotra R. Ecological optimization of irreversible Stirling and Ericsson heat engine cycles. *J Phys D Appl Phys* 2002;35:2668–75.
- [28] Veldhuizen DAV, Lamont GB. Multiobjective evolutionary algorithms: analyzing the state-of-the-art. *Evol Comput* 2000;8:125–47.
- [29] Konak A, Coit DW, Smith AE. Multi-objective optimization using genetic algorithms: a tutorial. *Reliab Eng Syst Saf* 2006;91:992–1007.
- [30] Bck T, Fogel D, Michalewicz Z. *Handbook of evolutionary computation*. Oxford Univ. Press; 1997.
- [31] Ahmadi MH, Hosseinzade H, Sayyaadi H, Mohammadi AH, Kimiaghdam F. Application of the multi-objective optimization method for designing a powered Stirling heat engine: design with maximized power, thermal efficiency and minimized pressure loss. *Renew Energy* 2013;60:313–22.
- [32] Ahmadi MH, Sayyaadi H, Mohammadi AH, Barranco-Jimenez MA. Thermo-economic multi-objective optimization of solar dish-Stirling engine by implementing evolutionary algorithm. *Energy Conversion and Management* 2013;73:370–80.
- [33] Sayyaadi H, Amlashi EH, Amidpour M. Multi-objective optimization of a vertical ground source heat pump using evolutionary algorithm. *Energy Convers Manage* 2009;50:2035–46.
- [34] Lazzaretto A, Toffolo A. Energy, economy and environment as objectives in multi-criterion optimization of thermal systems design. *Energy* 2004;29: 1139–57.
- [35] Sayyaadi H. Multi-objective approach in thermoenviromonic optimization of a benchmark cogeneration system. *Applied Energy* 2009;86:867–79.
- [36] Wang J, Yan Z, Wang M, Li M, Dai Y. Multi-objective optimization of an organic Rankine cycle (ORC) for low grade waste heat recovery using evolutionary algorithm. *Energy Convers Manage* 2013;71:146–58.
- [37] Taal M, Bulatov I, Kleme_s J., Stehlik P. Cost estimation and energy price forecasts for economic evaluation of retrofit projects. *Applied Thermal Engineering* 2003;23:1819–35.
- [38] Mazur V. Fuzzy thermoeconomic optimization of energy-transforming systems. *Appl Energy* 2007;84:749–62.
- [39] Olson DL. *Decision aids for selection problems*. New York: Springer; 1996.